Further Results of Flat Space-time Theory of Gravitation

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The anomalous acceleration of spacecrafts in the solar system is explained. An explanation of the observed superluminal velocities of jets at extragalactic objects is given. The extension of quasars can be larger as generally assumed, i. e. quasars must not be very compact objects. An explanation of the high energy loss per unit time of quasars is given. The relation between the velocity of an object in the universe and its redshift is stated. All these results are received from cosmological models studied by flat space-time theory of gravitation and the post-Newtonian approximation of perfect fluid in these cosmological models where clocks at earlier times are going faster than at present.

Key words: Flat Space-time Theory of Gravitation; Cosmological Models; Anomalous Acceleration; Superluminal Velocities; Extension and Energy Loss of Quasars.

1. Introduction

A covariant theory of gravitation in flat space-time [1] has been studied in several papers. Gravitational redshift, light deflection, perihelion precession, radar time delay, post-Newtonian approximation, gravitational radiation, and the precession of the spin axis of a gyroscope in the orbit of a rotating body agree with the corresponding results of general relativity to the accuracy needed by the experiment. Birkhoff's theorem is not valid for this theory of gravitation. Furthermore, the theory gives non-singular, homogeneous, isotropic cosmological models under natural assumptions. A summary of flat space-time theory of gravitation with the above mentioned applications can be found in [2], where references to the detailed studies are stated.

In this paper the theory of gravitation in flat space-time [1] is summarized. A homogeneous, isotropic, non-singular cosmological model [3] is stated. Entropy is produced in this model, and the space of any model in flat space-time theory of gravitation must be flat, as recently verified by observations. A perfect fluid in this cosmological model is considered [4], and the post-Newtonian approximation is stated.

The post-Newtonian approximation of a perfect fluid in the universe is used to derive further new results of flat space-time theory of gravitation which cannot be received by Einstein's general theory of relativity:

1. An explanation of the anomalous acceleration of spacecrafts in the solar system is given. This acceleration is opposite to the direction of the velocity of the object. In particular, it is in the direction to the sun for objects moving radially away from the sun. Anderson et al. [5] have studied the radio Doppler data of Pioneer 10 and 11, and they received an anomalous acceleration of these spacecrafts of about $8 \cdot 10^{-8}$ cm/s^2 in the direction to the sun. This effect was also confirmed by Markwardt [6]. Several explanations of the anomalous acceleration have been given but Anderson et al. [5] conclude that no theory can explain this effect. Recently two new explanations appeared by Scheffer [7] and Marmet [8], which are different from our explanation.

2. In the universe, flat space-time theory of gravitation gives locally the vacuum light velocity for any observer and for all times, but clocks at present time go slower than at earlier times. Therefore, for the observer at present time the light velocity at distant objects is larger than the local vacuum light velocity. This gives a new explanation of the observed superluminal velocities at extragalactic objects. This result was already stated in [3].

3. The remarks of point 2 also yield upper bounds of the extension of quasars, and they can be by several orders of magnitude larger than generally assumed, i. e. quasars must not be very compact objects.

4. The electro-magnetic field equations of an electric current four-vector in a gravitational field are de-

rived. The electro-magnetic radiation of charges moving in the gravitational field of a distant object in the universe is calculated. It follows that the observed high electro-magnetic energy loss per unit time of quasars is explained without the assumption of small distances of the moving charges to the object, i. e. quasars must not be very compact.

5. The connection between the velocity of an object in the universe moving away from the observer and its measured redshift is calculated.

The results of the points 1 to 5 were given on the meeting "Physical Interpretations of Relativity Theory IX" in London in September 2004 (see [9]).

2. Flat Space-time Theory of Gravitation

In this section a covariant theory of gravitation in flat space-time, studied in [1], is summarized. Subsequently, the sum is only taken over Greek letters, which run from 1 to 4. The line-element of the flat space-time metric is

$$(\mathrm{d}s)^2 = -\eta_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta \tag{2.1}$$

where (η_{ij}) is a symmetric tensor. In the special case of the pseudo-Euclidean metric it holds

$$(\eta_{ij}) = \mathrm{diag}(1,1,1,-1).$$

Here, x^1, x^2, x^3 are the Cartesian coordinates and $x^4 = ct$. Put

$$\eta = \det(\eta_{ij}).$$

The gravitational field is descibed by a symmetric tensor (g_{ij}). Let (g^{ij}) be defined by

$$g_{i\alpha}g^{\alpha j} = \delta_i^j, \quad g^{i\alpha}g_{\alpha j} = \delta_j^i$$

and put

$$G = \det(g_{ij}).$$

The proper time τ (atomic time) is given by

$$c^2(\mathrm{d}\tau)^2 = -g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta, \tag{2.2}$$

where $(\mathrm{d}\tau)^2 = 0$ holds for light rays and $\mathrm{d}\tau$ with $(\mathrm{d}\tau)^2 > 0$ is measured by atomic clocks. In (2.1) the used space-coordinates and the system time t are stated.

The covariant field equations for the gravitational field have the form (formally similar to Einstein's theory)

$$\tilde{R}_j^i - \frac{1}{2}\delta_j^i \tilde{R}_\alpha^\alpha = 4\kappa T_j^i \tag{2.3a}$$

with

$$\tilde{R}_j^i = \left[\left(\frac{-G}{-\eta}\right)^{1/2} g^{\alpha\beta} g_{j\nu} g^{\nu i}{}_{/\beta}\right]_{/\alpha}, \tag{2.3b}$$

where the bar "/" denotes the covariant derivative relative to the flat space-time metric, and T_j^i is the total energy-momentum tensor of the gravitational field, of matter, of the cosmological constant and of other fields such as the electro-magnetic field or a scalar field. The tensor of the gravitational field is given by

$$T^{\mathrm{G}}{}^i_j = \frac{1}{8\kappa}\left[\left(\frac{-G}{-\eta}\right)^{1/2} g_{\nu\mu} g_{\gamma\delta} g^{i\beta}\left(g^{\nu\gamma}{}_{/j} g^{\mu\delta}{}_{/\beta} - \frac{1}{2} g^{\nu\mu}{}_{/j} g^{\gamma\delta}{}_{/\beta}\right) + \frac{1}{2}\delta_j^i L_G\right] \tag{2.3c}$$

with $\kappa = 4\pi k/c^4$ (k: gravitational constant) and the Lagrangian

$$L_G = -\left(\frac{-G}{-\eta}\right)^{1/2} g_{\nu\mu} g_{\gamma\delta} g^{\alpha\beta}\left(g^{\nu\gamma}{}_{/\alpha} g^{\mu\delta}{}_{/\beta} - \frac{1}{2} g^{\nu\mu}{}_{/\alpha} g^{\gamma\delta}{}_{/\beta}\right). \tag{2.3d}$$

It is worth mentioning that Euler's equations of the Lagrangian (2.3d) give the field equations (2.3a) with (2.3b) and (2.3c) neglecting non-gravitational energy-momentum tensors.

Lowering [raising] of inidces is received by g_{ij} [g^{ij}] and taking the sum over corresponding indices, as e. g.

$$T_j^i = g_{j\alpha}T^{i\alpha}, \quad T^{ij} = g^{j\alpha}T^i{}_\alpha.$$

Furthermore, a covariant conservation law of the total energy-momentum holds:

$$T^\alpha_{i/\alpha} = 0. \tag{2.4}$$

The field equations together with the conservation law of the total energy-momentum tensor give four equations of motion for the matter.

The application of the theory of gravitation in flat space-time gives the same results as general relativity to the experimentally needed accuracy for the following effects: gravitational redshift, light deflection, perihelion precession, radar time delay, post-Newtonian approximation, gravitational radiation and the precession of a spin axis of a gyroscope.

Birkhoff's theorem does not hold for flat space-time theory of gravitation. Furthermore, the theory gives homogeneous, isotropic, non-singular cosmological models.

A summary of these results can be found in [2], where references to the different articles are given. It is worth mentioning that Rosen [10] was the first author who has considered Einstein's theory in a flat space-time metric. A flat space-time successive approximation procedure of Gupta [11] gives Einstein's theory. Kohler [12] started from a flat space-time metric with several suitable Lagrangians similar to our considerations. Some arguments against Kohler's theory of gravitation are given by Papapetrou et al. [13]. In the meantime many papers appeared with a background metric (2.1) and an additional quadratic form (2.2) called bimetric theories.

3. A Non-singular Cosmological Model

In this section a homogeneous, isotropic, non-singular cosmological model is stated. The detailed study is given in [3]. It is worth mentioning that the space must be flat for any cosmological model of flat space-time theory of gravitation, which was recently verified by astrophysical observations. The flat space-time metric of the universe is the pseudo-Euclidean geometry. The potentials are

$$(g_{ij}) = \mathrm{diag}(a^2(t), a^2(t), a^2(t), -1/h(t)), \quad (3.1)$$

where comoving coordinates are used, i.e. the four-velocity is given by

$$u^i = 0\,(i=1,2,3), \quad u^4 = c\frac{\mathrm{d}t}{\mathrm{d}\tau} \quad (3.2)$$

with

$$\mathrm{d}\tau = \mathrm{d}t/\sqrt{h(t)}. \quad (3.3)$$

The energy-momentum tensor in (2.3) consists of dust, radiation, additional matter with a stiff equation of state, i.e. $p_a = \rho_a$, cosmological constant and gravitation. Then, the field equations (2.3) give two coupled differential equations of order two for $a(t)$ and $h(t)$ of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\sqrt{h}\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\right) = 2\kappa c^4\left(\frac{1}{2}\rho_\mathrm{m} + \frac{1}{3}\rho_\mathrm{r} + \frac{\Lambda}{2\kappa c^2}\frac{a^3}{\sqrt{h}}\right),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\sqrt{h}\frac{1}{h}\frac{\mathrm{d}h}{\mathrm{d}t}\right) = 4\kappa c^4\left(\frac{1}{2}\rho_\mathrm{m} + \rho_\mathrm{r} + 2\rho_\mathrm{a} + \frac{1}{8\kappa c^2}L_G - \frac{\Lambda}{2\kappa c^2}\frac{a^3}{\sqrt{h}}\right)$$

where $\rho_i (i = \mathrm{m,r,a})$ denote the densities of dust (matter), radiation and additional matter.

The Langrangian is given by

$$L_G = \frac{1}{c^2}a^3\sqrt{h}\Bigg(-6\left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 + 6\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\frac{1}{h}\frac{\mathrm{d}h}{\mathrm{d}t} + \frac{1}{2}\left(\frac{1}{h}\frac{dh}{dt}\right)^2\Bigg).$$

The conservation law (2.4) inplies by integration

$$(\rho_\mathrm{m} + \rho_\mathrm{r} + \rho_\mathrm{a})c^2 + \frac{1}{16\kappa}L_G + \frac{\Lambda}{2\kappa}\frac{a^3}{\sqrt{h}} = \lambda c^2$$

where λ is a constant of integration.

It is worth mentioning that by virtue of the flat background metric (2.1) any transformation of the time would again give two coupled differential equations for two other functions, i.e. $h(t)$ is no gauge function.

As initial conditions let us use the present time $t = 0$. Then,

$$a(0) = h(0) = 1, \quad \dot{a}(0) = H_0, \quad \dot{h}(0) = \dot{h}_0, \quad (3.4)$$

where the dot denotes the t-derivative, H_0 is the Hubble constant and $\dot{h}_0$ is an additional constant which is zero for Einstein's theory. Let Ω_i $(i = \mathrm{m,r,a},\Lambda)$ be the usual definitions of dust (matter), radiation, additional matter and cosmological constant and define the two constants

$$K_0 = (\Omega_\mathrm{m} + \Omega_\mathrm{r} + \Omega_\mathrm{a} + \Omega_\Lambda - 1)/\Omega_\mathrm{m} \quad (3.5)$$

$$K_1 = (1 - \Omega_\mathrm{m} - \Omega_\mathrm{r} - \Omega_\Lambda)/\Omega_\mathrm{m}. \quad (3.6)$$

Then, a non-singular, cosmological model, i. e. all densities are finite for all times, yields

$$K_0 > 0, \tag{3.7}$$

and the second law of thermodynamics, i. e. entropy can not decrease for all times, gives

$$K_1 > 0. \tag{3.8}$$

Hence, it follows that $\Omega_a > 0$ is necessary. The presently assumed best cosmological parameters are

$$\Omega_m \approx 0.3,\ \Omega_r \ll \Omega_m,\ \Omega_\Lambda \approx 0.7,\quad H_0 \approx 70 \frac{\text{km}}{\text{sec Mpc}} \tag{3.9}$$

and let be

$$\Omega_a \ll 0.03, \tag{3.10}$$

then the age of the universe measured with proper time is given by

$$\tau(0) \approx \frac{1}{3\sqrt{\Omega_\Lambda} H_0} \log\left(1 + 2\frac{\Omega_\Lambda}{\Omega_m} + 2\frac{\sqrt{\Omega_\Lambda}}{\Omega_m}\right) \approx 13.5 \cdot 10^9 y. \tag{3.11}$$

An analytic solution can be given under the assumptions (3.9). Put

$$B = \Omega_m K_1 + \frac{1}{2}\Omega_m + (\Omega_m K_1)^{1/2}, \tag{3.12a}$$

$$F(t) = \exp\left(\sqrt{\frac{3K_1}{K_0}} \operatorname{arctg}\left(\frac{\sqrt{3\Omega_m K_0} H_0 t}{1 + \frac{1}{2}\frac{\varphi_0}{H_0} H_0 t}\right)\right), \tag{3.12b}$$

then, the solution of $a(t)$ has the form

$$a^3(t) = 2\Omega_m K_1 B F(t) \Big/ \left[\left(B - \frac{1}{2}\Omega_m F(t)\right)^2 - \Omega_m K_1 \Omega_\Lambda F^2(t)\right], \tag{3.12c}$$

and $h(t)$ follows from

$$a^3 h^{1/2} = 2\kappa c^4 \lambda t^2 + \varphi_0 t + 1, \tag{3.12d}$$

where

$$\varphi_0 = 3H_0\left(1 + \frac{1}{6}\frac{\dot{h}_0}{H_0}\right),$$

$$2\kappa c^4 \lambda = \frac{1}{4}{\varphi_0}^2 + 3{H_0}^2 \Omega_m K_0.$$

Let

$$0 < K_0 \ll K_1 \ll 1. \tag{3.13}$$

Then $a(t)$ starts from a small positive value at t equal to minus infinity and then increases for all t, whereas $h(t)$ starts from infinity, decreases to a small positive value and then increases to infinity as t goes to infinity. It is worth mentioning that $h(t)$ is increasing since the time where large redshifts are observed.

The solutions (3.12) are exact, but by virtue of their complexities the solutions are represented as functions of the redshift z. Then they can be better compared with the results of Einstein's theory. It holds for all redshifts z, as by general relativity

$$a(t) = 1/(1+z). \tag{3.14a}$$

We have for not too large z

$$h^{1/2}(z) = (1+z)^3\left[\left(\frac{3}{4}\frac{\varphi_0}{H_0} + \frac{3}{4}\frac{\varphi_0}{H_0}\left(1 + \Omega_m\left((1+z)^3 - 1\right)\right)^{1/2} - \frac{9}{4}\Omega_m\right) \Big/ \left(\left(\frac{1}{2}\frac{\varphi_0}{H_0}\right)^2\left((1+z)^3 - 1\right) + \frac{3}{2}\frac{\varphi_0}{H_0} - \frac{9}{4}\Omega_m\right)\right]^2. \tag{3.14b}$$

A galaxy at a redshift z has the distance

$$r = \frac{c}{H_0} \int_0^z \frac{\mathrm{d}x}{(\Omega_\Lambda + \Omega_\mathrm{m}(1+x)^3)^{1/2}} \tag{3.15}$$

for all observable redshifts. The formula (3.15) is identical with the result of Einstein's theory, and it gives the measured accelerated expansion of the universe by virtue of the cosmological constant. The differences to the results of Einstein arises in the beginning of the universe and it holds as $t \to -\infty$

$$a(t) = \tilde{a} \exp\left(\sqrt{\Omega_\mathrm{m} K_1} / \left(-\left(\frac{1}{2}\frac{\varphi_0}{H_0}\right)^2 H_0 t\right)\right),$$

where

$$\tilde{a} \approx (2\Omega_\mathrm{m} K_1 / B)^{1/3} \exp\left(-\pi\sqrt{K_! / (3K_0)}\right).$$

4. Newtonian Approximation in the Universe

The post-Newtonian approximation of a perfect fluid in the universe is studied in [4]. In this section the Newtonian approximation of a perfect fluid in the universe is stated, which is sufficient for the subsequent considerations.

Let

$$\rho(x,t),\, v(x,t) = (v^1(x,t),\, v^2(x,t),\, v^3(x,t))$$

be the density, resp. the three-velocity of the perfect fluid in the universe. Then, the potentials have the form

$$\begin{aligned} g_{ij} &= a^2\left(1 + \frac{2}{c^2}U\right), i = j = 1,2,3, \\ g_{ij} &= -\frac{1}{h}\left(1 - \frac{2}{c^2}U\right), i = j = 4, \\ g_{ij} &= 0, i \neq j \end{aligned} \tag{4.1}$$

with

$$U = k\frac{\sqrt{h}}{a} \int \frac{\rho(x',t)}{|x-x'|} \mathrm{d}^3x', \tag{4.2}$$

where $|\cdot|$ is the Euclidean norm. The density

$$\rho^* = \rho \frac{\mathrm{d}t}{\mathrm{d}\tau} \tag{4.3}$$

implies the conserved mass

$$M = \int \rho^*(x',t) \mathrm{d}^3x'. \tag{4.4}$$

The equations of motion in the universe to the Newtonian approximation have the form:

$$\begin{aligned} &\frac{\partial}{\partial t}\left(a^2\sqrt{h}v^i\right) + \sum_{\alpha=1}^{3} v^\alpha \frac{\partial}{\partial x^\alpha}\left(a^2\sqrt{h}v^i\right) \\ &= -\frac{1}{a\sqrt{h}} k \int \rho^*(x',t) \frac{x^i - x^{i'}}{|x-x'|^3} \mathrm{d}^3x'. \end{aligned} \tag{4.5}$$

5. Applications

5.1. Anomalous Acceleration

The equations of motion are rewritten:

$$\begin{aligned} &\frac{\partial v^i}{\partial t} + \sum_{\alpha=1}^{3} v^\alpha \frac{\partial v^i}{\partial x^\alpha} + \left(2\frac{\dot{a}}{a} + \frac{1}{2}\frac{\dot{h}}{h}\right) v^i \\ &= -\frac{1}{a^3 h} k \int \rho^*(x',t) \frac{x^i - x^{i'}}{|x-x'|} \mathrm{d}^3x'. \end{aligned}$$

These equations are applied to a finite number of point particles ℓ, i. e. the sun, the planets and the spacecraft with velocities $v_\ell = (v_\ell^1, v_\ell^2, v_\ell^3)$ and the conserved masses

$$M_\ell = \int \rho_\ell^*(x',t) \mathrm{d}^3x'. \tag{5.1}$$

It follows with the initial conditions (3.4) at present time under the assumption

$$0 < H_0 \ll \dot{h}_0 \tag{5.2}$$

in the solar system for the particle ℓ

$$\frac{\mathrm{d}v_\ell}{\mathrm{d}t} = -\frac{1}{2}\dot{h}_0 v_\ell - k \sum_{j \neq \ell} \frac{M_j(x_\ell - x_j)}{\left|x_\ell - x_j\right|^3}. \tag{5.3}$$

The left hand side is the acceleration of the particle ℓ, which is given by the Newton law of all particles acting on particle ℓ and the anomalous acceleration

$$\Delta \frac{\mathrm{d}v_\ell}{\mathrm{d}t} = -\frac{1}{2}\dot{h}_0 v_\ell. \tag{5.4}$$

Hence, the anomalous acceleration of any object is opposite to the velocity of the object.

Anderson et al. [5] have measured for Pioneer 10 with a distance of about 67 AU from the sun a nearly constant velocity of 12.2 km/s away from the sun and an anomalous acceleration of about $8.74 \cdot 10^{-8}$ cm/s^2. Hence, relation (5.4) gives

$$\dot{h}_0 \approx 1.4 \cdot 10^{-13}\, 1/\mathrm{s}. \tag{5.5}$$

It follows by the use of the Hubble constant

$$\dot{h}_0 / H_0 \approx 60,000,$$

justifying assumption (5.2). Relation (5.4) with (5.5) holds at present time for any object in the solar system.

It is worth mentioning that the functions $a(t)$ and $h(t)$ describing the universe are now uniquely defined by the knowledge of the density parameters, the Hubble constant H_0 and the constant $\dot{h}_0$.

We get at present time

$$\mathrm{d}t = \mathrm{d}\tau, \tag{5.6}$$

i. e. the observer uses the present atomic clocks.

It is well known that the trajectory of any object is independent of the time used. Therefore, let us introduce the proper time τ instead of the abolute time (system time) t and the corresponding velocities

$$\tilde{v}^i = \frac{\mathrm{d}x^i}{\mathrm{d}\tau} = \frac{\mathrm{d}x^i}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\tau} = v^i \sqrt{h}. \tag{5.7a}$$

Then, the equations of motion (4.5) are rewritten:

$$\begin{aligned} &\frac{\partial}{\partial \tau}(a^2 \tilde{v}^i) + \sum_{a=1}^{3} \tilde{v}^a \frac{\partial}{\partial x^a}(a^2 \tilde{v}^i) \\ &= -\frac{1}{a} k \int \rho^*(x', t) \frac{x^i - x^{i'}}{x - x'} \mathrm{d}^3 x'. \end{aligned} \tag{5.7b}$$

It follows analogously to the previous considerations for the point particles

$$\frac{\mathrm{d}\tilde{v}_\ell}{\mathrm{d}\tau} = -k \sum_{j \neq \ell} \frac{M_j (x_\ell - x_j)}{|x_\ell - x_j|^3}. \tag{5.8}$$

This is the well known Newton law for the particle ℓ. The anomalous acceleration does not influence the trajectory of any object. Therefore, the orbits of the planets are given by Newton's law in agreement with the results for Earth and Mars, the orbits of which are known to high accuracy.

5.2. Superluminal Velocities

Let us introduce, instead of the comoving coordinates (x^1, x^2, x^3), the measured coordinates $(\tilde{x}^1, \tilde{x}^2, \tilde{x}^3)$ at the time t by

$$\tilde{x} = a(t) x^i, \quad (i = 1,2,3). \tag{5.9}$$

Then, the light velocity at time t for an observer at present time is given by

$$\left| \frac{\mathrm{d}\tilde{x}}{\mathrm{d}t} \right| = c / \sqrt{h(t)}. \tag{5.10}$$

Eckart et al. [14] have considered the quasar 1928+738 with redshift $z = 0.36$, and they observed subcomponents with transverse velocities relative to the quasar of $13.85c$. Then, the real velocity on the jet with the angle β between the line of sight and the velocity is $13.85c / \sin(\beta)$.

But the light velocity (5.10) at the quasar for the present observer is

$$\left| \frac{\mathrm{d}\tilde{x}}{\mathrm{d}t} \right| \approx 19 \cdot 10^6 c$$

which is much larger than the measured velocity of the jet even for small β.

But the local light velocity for any observer in the universe measured with his atomic clock is, for all times, given by

$$\left| \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tau} \right| = c. \tag{5.11}$$

5.3. Extension of Quasars

Quasars are varying their brightness on small time scales (see e. g. Begelman et al. [15]) which implies an upper bound of the extension of these objects. Some objects have a variability of only some hours. Let Δt be the time scale, then an upper bound of the extension is

$$\mathrm{d} \leq \left| \frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} \right| \Delta t = c \Delta t / \sqrt{h(t)}. \tag{5.12}$$

Hence, the upper bound can be, depending on the redshift, several orders of magnitude larger than the standard upper bound $c\Delta t$, i. e. quasars must not be as compact as generally assumed.

5.4. Electro-magnetic Radiation

The Lagrangian of the electro-magnetic field in a gravitational field has the form

$$L_{\mathrm{E}} = \frac{1}{4}\left(\frac{-G}{-\eta}\right)^{1/2} g^{\alpha\beta} g^{\nu\mu} F_{\alpha\nu} F_{\beta\mu} + A_\alpha J^\alpha, \quad (5.13)$$

where (J^1, J^2, J^3, J^4) is the electric current four-vector, $A_i (i = 1,2,3,4)$ are the electro-magnetic potentials and

$$F_{ij} = A_{j/i} - A_{i/j} \quad (5.14)$$

are the electro-magnetic field strengths. The Lagrangian (5.13) gives by Euler's differential equations the field equations

$$\left(\left(\frac{-G}{-\eta}\right)^{1/2} g^{i\mu} g^{\alpha\nu} F_{\nu\mu}\right)_{/\alpha} = J^i (i = 1,2,3,4). \quad (5.15)$$

We get by the use of (5.14)

$$F_{ij/k} + F_{jk/i} + F_{ki/j} = 0. \quad (5.16)$$

The relations (5.15) and (5.14) imply the conservation of charge:

$$J^\alpha{}_{/\alpha} = 0. \quad (5.17)$$

(5.15) and (5.16) are the electro-magnetic field equations in covariant form. The energy-momentum tensor of the electro-magnetic field also follows from the Lagrangian, and it holds

$$T^{\mathrm{E}\,i}{}_j = \left(\frac{-G}{-\eta}\right)^{1/2} \Big(g^{i\beta} g^{\nu\mu} F_{j\nu} F_{\beta\mu} - \frac{1}{4}\delta^i_j g^{\alpha\beta} g^{\nu\mu} F_{\alpha\nu} F_{\beta\mu}\Big). \quad (5.18)$$

It is worth mentioning that the Lagrangian (5.13) gives the correct equations for the electro-magnetic field (5.15) and the correct electro-magnetic energy-momentum tensor (5.18) of this field.

The field equations can be rewritten by the use of the classical derivatives in the more conventional form

$$\frac{1}{\sqrt{-\eta}} \frac{\partial}{\partial x^\alpha}\left(\sqrt{-G} g^{i\mu} g^{\alpha\nu} F_{\nu\mu}\right) = J^i, \quad (5.19)$$

$$\frac{\partial}{\partial x^k} F_{ij} + \frac{\partial}{\partial x^i} F_{jk} + \frac{\partial}{\partial x^j} F_{ki} = 0 \quad (5.20)$$

with

$$\frac{1}{\sqrt{-\eta}} \frac{\partial}{\partial x^\alpha}\left(\sqrt{-\eta} J^\alpha\right) = 0. \quad (5.21)$$

The equations (5.19) are considered in the universe which has been described in Chapter 3. Then, the equation (5.19) gives the relations

$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^\alpha}\left(\frac{1}{a\sqrt{h}} F_{\alpha i}\right) - \frac{\partial}{\partial ct}\left(a\sqrt{h} F_{4i}\right) = J^i, \quad (i = 1,2,3),$$
$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^\alpha}\left(a\sqrt{h} F_{4\alpha}\right) = J^4. \quad (5.22)$$

Let us introduce the potentials A_i instead of the field strength and let us assume the gauge condition

$$\frac{1}{a\sqrt{h}} \sum_{\alpha=1}^{3} \frac{\partial}{\partial x^\alpha} A_\alpha - \frac{\partial}{\partial ct}\left(a\sqrt{h} A_4\right) = 0, \quad (5.23)$$

then, the field equations (5.22) can be rewritten:

$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^\alpha}\left(\frac{1}{a\sqrt{h}} \frac{\partial A_i}{\partial x^\alpha}\right) - \frac{\partial}{\partial ct}\left(a\sqrt{h} \frac{\partial A_i}{\partial ct}\right) = J^i, \quad (i = 1,2,3),$$
$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^\alpha}\left(a\sqrt{h} \frac{\partial A_4}{\partial x^\alpha}\right) - a\sqrt{h} \frac{\partial}{\partial ct}\left(a\sqrt{h} \frac{\partial}{\partial ct}\left(a\sqrt{h} A_4\right)\right) = -J^4. \quad (5.24)$$

A wave solution of (5.24) is studied.

Assume that at time t_{e} a wave is emitted from an object at distance x' and put

$$u(x, x', t) = t_{\mathrm{e}} + \int_{t_{\mathrm{e}}}^{t} \frac{\mathrm{d}t}{a\sqrt{h}} - |x - x'|/c. \quad (5.25)$$

Then, the retarded solutions have the form

$$A_i(x,t) = -\frac{1}{4\pi} \int \frac{J^i(u(x,x',t))}{|x - x'|} \mathrm{d}^3 x', \quad (i = 1,2,3),$$

$$A_4(x,t) = \frac{1}{a\sqrt{h}} \frac{1}{4\pi} \int \frac{J^4(u(x,x',t))}{|x - x'|} \mathrm{d}^3 x'. \quad (5.26)$$

The covariant conservation law of charge (5.21) implies the gauge condition (5.23).

The densities of the energy and the energy current in the universe have the form

$$T^{\mathrm{E}}{}^{4}_{4} = -\frac{1}{2}\left(a\sqrt{h}\sum_{\alpha=1}^{3}(F_{4\alpha})^2 + \frac{1}{2}\frac{1}{a\sqrt{h}}\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}(F_{\alpha\beta})^2\right),$$

$$T^{\mathrm{E}}{}^{i}_{4} = \frac{1}{a\sqrt{h}}\sum_{\alpha=1}^{3}F_{i\alpha}F_{4\alpha}, \quad (i=1,2,3). \quad (5.27)$$

The conservation law of the energy for the pseudo-Euclidean geometry in the exterior of the current four-vector satisfies the relation

$$\frac{\partial}{\partial ct}\left(-T^{\mathrm{E}}{}^{4}_{4}\right) = \sum_{\alpha=1}^{3}\frac{\partial}{\partial x^\alpha}T^{\mathrm{E}}{}^{\alpha}_{4}. \quad (5.28)$$

The results (5.26) with (5.25) are applied to a point charge with density $\rho_c(x,t)$ and the trajectory $x(t) = \left(x^1(t), x^2(t), x^3(t)\right)$, i. e.

$$J^i = \frac{1}{c}\rho_c\frac{\mathrm{d}t}{\mathrm{d}\tau}\delta(x-x(t))\frac{\mathrm{d}x^i}{\mathrm{d}t}, \quad (i=1,2,3,4).$$

The conservation law of the charge gives

$$Q = \int \rho_c(x',t)\frac{\mathrm{d}t}{\mathrm{d}\tau}\mathrm{d}^3x'.$$

Then, the electro-magnetic potentials are

$$A_i(x,t) = -\frac{Q}{4\pi}\frac{1}{c}\frac{\mathrm{d}x^i(u(x,x(t),t))}{\mathrm{d}u}\frac{1}{|x-x(t)|}, \quad (i=1,2,3),$$

$$A_4(x,t) = \frac{1}{a(t)\sqrt{h(t)}}\frac{Q}{4\pi}\frac{1}{|x-x(t)|}.$$

The potentials and their derivatives must be calculated at the position of the observer, i. e. $x=0$, and at the present time $t=0$. By virtue of $|x(0)| \approx |x(t_{\mathrm{e}})|$ we have

$$u(0,x(0),0) = t_{\mathrm{e}} + \int_{t_{\mathrm{e}}}^{0}\frac{\mathrm{d}t}{a\sqrt{h}} - \frac{|x(0)|}{c} \approx t_{\mathrm{e}}.$$

Therefore, it follows for the present observer, i. e. $x=0$ at the present time for $i,j=1,2,3$

$$\frac{\partial A_i(0,0)}{\partial x^j} = -\frac{Q}{4\pi}\frac{1}{c^2}\frac{\mathrm{d}^2x^i(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2}\frac{x^j(0)}{|x(0)|^2} + \mathrm{cor}$$

$$\frac{\partial A_i(0,0)}{\partial x^4} = -\frac{Q}{4\pi}\frac{1}{c^2}\frac{\mathrm{d}^2x^i(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2}\frac{1}{|x(0)|} + \mathrm{cor}$$

$$\frac{\partial A_4(0,0)}{\partial x^i} = \mathrm{cor}$$

with $\mathrm{cor} = O\left(1/|x(0)|^2\right)$. Let the distant object be in the direction x^1 with the distance d, i. e.

$$d = x^1(0) = |x(0)|, \quad x^2(0) = x^3(0) = 0,$$

then, the relations imply for $i=1,2,3$:

$$\frac{\partial A_i(0,0)}{\partial x^1} = \frac{\partial A_i(0,0)}{\partial x^4} = -\frac{Q}{4\pi c^2}\frac{\mathrm{d}^2x^i(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2}\frac{1}{d}$$

up to the order $O(1/d)$. All the other derivatives are of the order $O(1/d^2)$. Hence, we get up to the order $O(1/d)$

$$F_{i4}(0,0) = \frac{Q}{4\pi c^2}\frac{\mathrm{d}^2x^i(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2}\frac{1}{d}, \quad (i=1,2,3)$$

$$F_{i1}(0,0) = \frac{Q}{4\pi c^2}\frac{\mathrm{d}^2x^i(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2}\frac{1}{d}, \quad (i=2,3).$$

Therefore, the energy loss per unit surface and per unit time in the direction to the observer is by virtue of (5.28) up to the order $O(1/d^2)$

$$T^{\mathrm{E}}{}^{1}_{4} = \left(\frac{Q}{4\pi c^2}\right)^2\sum_{\alpha=2}^{3}\left(\frac{\mathrm{d}^2x^\alpha(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2}\right)^2\frac{1}{\mathrm{d}^2}. \quad (5.29)$$

Let us assume that the point charge is moving in a gravitational field of a mass M. Then, we have for $i=1,2,3$

$$\frac{\mathrm{d}^2x^i(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2} = -\frac{1}{(a^3h)(t_{\mathrm{e}})}kM\frac{x^i(t_{\mathrm{e}}) - x_0{}^i}{|x(t_{\mathrm{e}}) - x_0|^3}$$

where $x_0 = (x_0{}^1, x_0{}^2, x_0{}^3) = (d,0,0)$ is the center of the mass. The essential expression for the energy loss (5.29) is given by

$$\sum_{\alpha=2}^{3}\left(\frac{\mathrm{d}^2x^\alpha(t_{\mathrm{e}})}{\mathrm{d}t_{\mathrm{e}}^2}\right)^2 = (kM)^2\frac{\sum_{\alpha=2}^{3}\left[\left(a^{3/2}h^{1/2}\right)(t_{\mathrm{e}})\left(x^\alpha(t_{\mathrm{e}}) - x_0{}^\alpha\right)\right]^2}{\left[\left(a^{3/2}h^{1/2}\right)(t_{\mathrm{e}})\,|x(t_{\mathrm{e}}) - x_0|\right]^6},$$

i. e. every component is multiplied with the factor $(a^{3/2}h^{1/2})(t_{\mathrm{e}})$. Let r_{st} denote the standard distance where $a=h=1$, then, the real distance of the moving charge to the center of the mass is

$$r = |x(t_{\mathrm{e}}) - x_0| = \frac{1}{(a^{3/2}h^{1/2})(t_{\mathrm{e}})}r_{\mathrm{st}}. \quad (5.30)$$

Therefore, the real distance can be, depending on the redshift, several orders of magnitude larger than the standard distance.

The high energy loss of quasars per unit time is by standard considerations explained by ionized gas moving spirally to the center of a compact object (black hole). Since the distance (5.30) of the moving charge is several orders of magnitude larger than the standard distance, the ionized gas can be far away from the center of the object to produce the high energy loss per unit time.

This result agrees with the considerations of chapter 5.3 about the extension of quasars.

5.5. Redshift and Velocity

The velocity of stars or gas in galaxies is not directly measured, but their velocity is calculated from the redshift of these objects by a standard formula. It is shown in chapter 5.2 that for an observer at the present time the light velocity at a distant object is different from the vacuum light velocity. Therefore, new considerations on the observed redshift of a moving object and its calculated velocity are necessary. Let us assume that at the time t_e an atom in a distant object in the universe emits a light ray with energy E and momentum p_1 to the observer. Then by Chapt. 3

$$p_1 = -a(t_e)\sqrt{h(t_e)}E/c.$$

In the universe the energy $E(t)$ of the photon is varying with time and it holds ([16], (5.2))

$$E(t) = E/\left(a(t)\sqrt{h(t)}\right).$$

Let us now assume that the distant object is moving with the velocity $(v,0,0)$ then it follows for the emitted energy $\tilde{E}$ of the moving atom and the energy E of the same atom at rest ([17], (5.4) and (5.3))

$$\tilde{E}/c = E/c\frac{\partial x^4}{\partial \tilde{x}^4} + p_1\frac{\partial x^1}{\partial \tilde{x}^4} = \gamma\left(1 + a(t_e)\sqrt{h(t_e)}\frac{v}{c}\right)E/c$$

with

$$\gamma = \left(1 - a^2(t_e)h(t_e)\left(\frac{v}{c}\right)^2\right)^{-1/2}.$$

Hence, we get

$$E(t) = \gamma^{-1}\left(1 + a(t_e)\sqrt{h(t_e)}\frac{v}{c}\right)^{-1}\tilde{E}/\left(a(t)\sqrt{h(t)}\right).$$

It is shown ([16], (5.4)) that for an atom at rest, i. e. $v = 0$:

$$\tilde{E} = E(0) = E_0 a(t_e),$$

where E_0 is the energy emitted from the same atom at the present time. Therefore, we have

$$E(t) = \gamma^{-1}\left(1 + a(t_e)\sqrt{h(t_e)}\frac{v}{c}\right)^{-1}E_0\frac{a(t_e)}{a(t)\sqrt{h(t)}}. \tag{5.31}$$

Hence, the observer receives from the moving atom the emitted light ray with the energy

$$E(0) = \gamma^{-1}\left(1 + a(t_e)\sqrt{h(t_e)}\frac{v}{c}\right)^{-1}E_0 a(t_e), \tag{5.32}$$

i. e. the total redshift z is given by

$$\begin{aligned} z &= \frac{E_0}{E(0)} - 1 \\ &= \frac{1}{a(t_e)}\left(\frac{1 + a(t_e)\sqrt{h(t_e)}\frac{v}{c}}{1 - a(t_e)\sqrt{h(t_e)}\frac{v}{c}}\right)^{\frac{1}{2}} - 1 \qquad (5.33) \\ &\approx \frac{1}{a(t_e)} - 1 + \sqrt{h(t_e)}\frac{v}{c} = z_1 + z_2, \end{aligned}$$

where Taylor expansion is used. The redshift

$$z_1 = \frac{1}{a(t_e)} - 1 \tag{5.34}$$

states the expansion of space and

$$z_2 = \sqrt{h(t_e)}\frac{v}{c} = v/\left(c/\sqrt{h(t_e)}\right) \tag{5.35}$$

is the redshift of the moving object in the universe. (5.35) shows that the velocity of the moving object can be several orders of magnitude larger than that given by the standard formula, in agreement with the considerations of chapter 5.2 on superluminal velocities. Furthermore, (5.35) states the standard formula that the redshift is received by dividing the velocity of the object by the light velocity at this object (see Chapter 5.2).

Summarizing, it follows for flat space-time theory of gravitation in contrast to Einstein's general theory of relativity that clocks at earlier times, i. e. at distant objects, are going faster than those at present time, implying all the results of Chapter 5.

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